

This Chapter discusses verbal models in empirical science. In that context they are logically consistent by definition.<sup>a</sup> However, they will be treated within the context of two other language games just as relevant to Architecture: design and management, (see page 446) where integration and urgency are more important than logical consistency. That is the reason why differences in emphasis on consistency will be discussed as well. Consistency seems to necessitate incompleteness. For analysis this is less dangerous than for synthesis.

Consistency is denoted here by the formal logical model. With this, the relationship of verbal models with reality (reliability) is coming to the fore in the sense of truth or non-truth. However, on a different level incompleteness is remaining a special form of non-truth (half-truth). This demonstrates the restricted contribution of formal logic to the designing of models during the originating stages, when their consistency does not exist as yet, but must be made. That is a different language game. However, this does not detract at all from the importance of formal logic in the discussion of the still varying (increasing and decreasing) consistency aimed at whilst designing. It does not detract at all from the importance of formal logic during the evaluation of the design as soon as it is available in all its completeness. Also in that case the question is raised whether inconsistency is so ‘dangerous’. That is a language game as well. One should never forget that a crystal can not grow without a dislocation in its grid.

Next, the subject of causal consistency comes to the fore obviously. However, this form of consistency will be discussed later in the Chapter ‘Forecasting and Problem Spotting’ (see page 253). The question of incompleteness will get on the agenda again there, then explicitly in the sense of ‘*ceteris paribus*’.

## 23.1 LANGUAGE GAMES

In architecture three distinct language games<sup>b</sup> occur: those of designers, scholars and decision makers (respectively orientated on ‘being able’, ‘being knowledgeable’ and ‘being decisive’). The utterances of these agents in the building process (respectively ‘possible’, ‘true’ or ‘binding’ or not) can not be expressed completely in one another’s language, even when they are using the same words. By this they are causing linguistic confusions hard to disentangle, addressing respectively possible, probable and desirable futures. With it, temporal-spatial completeness, logical consistency and public urgency are becoming topics, respectively.

Grammatically ‘verbs of modality’<sup>c</sup> are reflecting the opinion of the speaker on the relationship to reality of what he is saying: possible (‘can’, ‘may’), probable (‘will’, ‘must’, ‘let’) and desirable (‘will’, ‘must’, and ‘may’; the latter two in a different sense). The language games introduced by this type of verbs are based on different reduction of imaginative realities.

The primary language of design is pictorial. When the designer records the key to symbols (legend) of his drawing, for instance red for urban areas, yellow for agriculture, and blue for water, he reduces the variation within the urban area, agriculture and the water. If he makes his drawing with pre-supposed legend unities, he first selects their site and form (state of dispersion) roughly and subsequently more precisely. So, during the design process he reduces further the tolerances of the design for the benefit of its feasibility.

The empirical researcher reduces reality in more abstract variables (set of related differences), but does not accept that a variable may assume any arbitrary value. He looks for functions between variables to restrict them in their freedom of change in order to make more precise predictions.

The policy maker reduces the problems to a few items on the agenda and tries to reach consensus by arrangements and appointments.

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Language games:	being able	knowing	selecting
Modes:	possible	probable	desirable
Sectors:	technique	science	administration
Activities:	design	research	policy
Reductions as to:			
Character:	legend	variable	agenda
Space or time:	tolerance	relations	appointments

181 Modal language games

a Nauta, D. (1970) *Logica en model*.  
 b Wittgenstein, L. (1953) *Philosophische Untersuchungen*. Recent edition: Wittgenstein, L. and G.E.M. Anscombe (1997) *Philosophical investigations*.  
 c Toorn, M.C. van den (1977) *Nederlandse Grammatica*, p. 91

### 23.2 MODALITIES

The empirical researcher is using, speaking strictly, exclusively logically consistent models, constructed from well defined concepts and variables.

Already in preparing the legends of his design the designer is disregarding components that do not belong to the legend-unit chosen strictly speaking (like parks in a city, green in red); or he is doing the opposite: over-emphasising details opening up possibilities. Furthermore, the designer is using them during his discourse in a variable significance in order to create intellectual space for the designing. To an empirical scholar the language of the designer is then ambiguous, or poly-interpretable and suggestive. The terms 'red', 'green' and 'blue' are already as variables not well defined, but they are more complete, also given the future possibilities. Utterances on the possible worlds of the design belong partially to 'modal logic', not discussed here.

The agenda of the policy maker is extremely incomplete by necessity. That is the reason why it is an art to get a topic 'on the agenda' of a meeting.

### 23.3 CHANGE OF ABSTRACTION

If the subject of a sentence is a concrete, touchable, visual, audible reality, the utterance is belonging to the concrete object language; in other cases to a more abstract meta-language (a distinction taken from the logic of classes).<sup>a</sup> Speaking about utterances, as happens in logic and in almost all sentences of the present Chapter, is belonging to a meta-language. This distinction is pre-empting paradoxes occurring if object language and meta-language are mixed within one full-sentence (change of abstraction, see page 37) as in the full-sentence 'What I am saying is a lie'.<sup>b</sup> In written language this is indicated by quotation marks. The change of abstraction may then be indicated by: 'What I am saying' now is a lie. In its turn the meta-language is layered, for if one is talking about logic, one is talking about a meta-language. One is finding oneself then in a high class of abstraction. The object language is layered as well, since one can talk on objects of a different scale, particularly in urban architecture. Changing of scale in a line of reasoning may lead to paradoxes; just think of 'inside' and 'outside', respectively on the levels of room, house, city). This is pre-empted by articulation of scale, as explained on page 37.

### 23.4 VERBS

Next to verbs of modality (can, shall, may must, will, let be to, dare to, serve to, need to, promise to, threaten to) there are countless independent verbs that may be 'steered' by them ('This building can collapse | has sagged | is being repaired'). By preceding such verbs of modality they can be harnessed for a specific language game (possible, desirable, probable).

Independent verbs are always pointing to a working (a function) or to a 'property' as a result thereof (for instance: 'This building sags'). When the full-sentence is employed in the language game of empirical study a subject from the 'existing' reality is described. It is also possible then to speak of 'models'.

### 23.5 CONSTRUCTING STATEMENTS

A full-sentence like 'This building is a cube' is providing a (always incomplete) description (predicate) of a subject<sup>c</sup> (in this case in object language a building that may be pointed at). This full-sentence establishes through the verb 'is' a relationship between this special building and more general, compressed earlier experiences ('cube' as an empirical concept of a lower class than the corresponding abstract geometrical concept).<sup>d</sup> 'Buildings are rectangular' is a description of all buildings with, in addition, a more general predicate than 'cube'. What is pronounced in it is not the case, as we know. The world is everything that is the case.<sup>e</sup> Language is also comprising negation of what is the case, and is pre-supposing the capability to imagine; it is possible to speak about what is not the case.

a Whitehead, A.N. and B. Russell (1910) *Principia mathematica*.

b A variant of Emimedes' paradox: 'All Cretans lie, said the Cretan'. If he lies, he speaks the truth; and vice-versa.

c Note, that the building as a real object outside of the sentence is acting as a subject within the sentence. This inversion is characteristic for each form of abstraction.

d Con-cept is Latin for 'taking together'.

e The first proposition of Wittgenstein, L. (1922) *Tractatus logico-philosophicus*. Recent edition: Wittgenstein, L., Pears D.F. et al. (2001) *Tractatus logico-philosophicus*.

A full-sentence always consists of subject and predicate. An utterance is a full-sentence that is the case or not. A design, an order or a vague, ambiguous full-sentence is, for instance, no utterance. Predicate logic is studying the internal construction of utterances and proposition logic their conjoining into assertions (propositions).

### 23.6 CONJOINING INTO ASSERTIONS

If more predicates are referring to the subject, one must conjoin with words such as ‘and’, ‘not and’, ‘or’, ‘neither...nor’, ‘if...then’ single utterances into an assertion:

- ‘This building is a cube *and* (this building) is rectangular’
- ‘This building is a cube *or* rectangular’
- ‘This building is *neither* a cube *nor* rectangular’
- ‘*If* this building is a cube, *then* it is rectangular’

‘If a building is a cube, then it is rectangular’ is always true, even if the building we are pointing at is no cube, and even if it is not rectangular.<sup>a</sup> Words that are composing utterances into an assertion, such as ‘if...then’, ‘and’ ‘or’, ‘nor’ can make the assertion they are composing become true, even if not all parts of the assertion are the case.<sup>b</sup> Proposition logic is studying this truth-determining operation.

### 23.7 COMPOSING LINES OF REASONING

In their turn, assertions may be composed into a line of reasoning by drawing a conclusion from premises. In contrast with utterances in an assertion, all premises in a line of reasoning must be true in order to draw a correct conclusion. The other way around, correctness of a conclusion is not assured even if all the premises are true.

In the following example the premises (above the dotted line) may be true, but the conclusion (below the line) is not valid.

- ‘If this building is a cube, then it is rectangular’
- ‘This building is rectangular’
- so
- ‘This building is a cube’

This line of reasoning can not be endorsed: purely on the ground of its structure, independent from our experience with cubes and straight angles. However, it is useful in the modality of what is possible. Then the conclusion must be: ‘This building may be a cube’: then it is valid again. The line of reasoning is also valid when the last premise is inter-changed with the conclusion:

- ‘If this building is a cube, then it is rectangular’
- ‘This building is a cube’
- so
- ‘This building is rectangular’

A line of reasoning with two premises and one conclusion, is known as a syllogism. A line of reasoning from general to particular is deductive, from particular to general inductive.

An inductive line of reasoning is not valid if the set of premises does not comprise all cases. One can only draw the conclusion that all buildings are rectangular, if one has checked all buildings, while observing for each building: ‘This building is rectangular’. There are then as many premises as there are buildings. It is only then that one can draw by complete induction the general conclusion that all buildings are rectangular. Yet, this completeness is virtual. What to do when one is finding buildings in a linguistic environment where the concept ‘rectangular’ does not exist, or is starting to apply at a certain length of both legs of the straight angle?

Study of the structure and validity of lines of reasoning, independent of their meaning (semantics) is the classical aim of logic (argumentation theory).<sup>c</sup> This entails, in a sequence of a decreasing complexity:

a Note, that ‘being the case’ relates to parts of the statement and ‘being true’ to its totality.  
 b We only talk about (un)truth when talking about statements. (Un)truth is, therefore, always a term from a meta-language.  
 c See: Eemeren, F.H. van (1996) *Fundamentals of argumentation theory, a handbook of historical backgrounds and contemporary developments*. Dutch translation: Eemeren, F.H. van, R. Grootendorst et al. (1997) *Handboek argumentatietheorie, historische achtergronden en hedendaagse ontwikkelingen*.

- set theory;
- modal logic (language games, modalities);
- class logic (level of abstraction);
- argumentation theory (lines of reasoning);
- proposition logic (assertions), and:
- predicate logic (utterances).

We are starting unconventionally with the smallest unit, the singular utterance, and within it the predicate and within that the full-sentence function.

### 23.8 FULL-SENTENCE FUNCTIONS AND FULL-SENTENCES

In predicate logic the structure of some assertions discussed here is usually rendered as follows (read for x ‘this building’):

<i>Formula:</i>	<i>Read:</i>
$K(x)$	being a cube as a working (function) of x.
$R(x)$	being rectangular as a working (function) of x.
$\exists x:K(x)$	there exists a x, ( $\exists x$ ), for which it is valid that (:) it is cubic ( $K(x)$ ).
$\forall x:R(x)$	for each x ( $\forall x$ ) is valid that (:) x is rectangular.
$\forall x:(K(x) \Rightarrow R(x))$	for each x ( $\forall x$ ) is valid that (:) if x is cubic, x is rectangular as well.

182 Operations with full-sentence functions

$K(x)$  and  $R(x)$  are full-sentence functions, names for a working of their argument (x), but not yet full-sentences themselves. The full-sentence functions are lacking a verb that the working of the argument, possibly on an object, is operationalising. Full-sentence functions are predicates without a verb. In addition a full-sentence is in need of a subject, an instancing of the argument (for example x: = this building).

In the language game of the designer full-sentence functions like villa(landscape) – ‘villa as a working of the landscape’ -, or landscape(villa)’- ‘landscape as working of the villa’ are operationalised only by the design. The working itself is not made explicit with a verb, unless it may be termed a design act. Often verbs like that do not exist; their existence is just suggested by the full-sentence function. In addition only the object of the predicate has been named, so that of the working just the object of operating has been named. These full-sentence functions are so useful particularly in this language game since just the direction of the working between the subject and the object is recorded.

In the language game of policy one is waiting for the verdict of a judge or decision of the board. The relation victim(suspect) must be made by juridical investigation in order to come to a ruling. A policy agenda must be become operational in agreements.

In empirical sciences it is precisely the trick to find for such a full-sentence function a formula or (weaker) a formulation, that is making it operational. Exact mathematical operationalising of a full-sentence function is called modelling.

However, a full-sentence function is in empirical study very useful for a function that has as yet not been made explicit in the problem formulation and the forming of a hypothesis; since assertions are in them not yet expected. For instance, one may surmise that the number of buildings or their volume G is a dependent variable of the population variables p and their prosperity w, considered to be independent for the time being. This working is readily noted as a full-sentence function:  $G(p,w)$ . In that case the problem formulation is ‘To which degree and how is G dependent on p en w?’ A hypothesis that has become operational may read:  $G(p,w) = p*w$ . The operator (\*) makes the working explicit; the full-sentence function has become a function.

### 2.9 FUNCTIONS

A full-sentence function becomes a function, when that function  $K( )$  or  $R( )$  has been made explicitly ‘operational’ (e.g.: ‘ $K( )$  := being a cube.’ Or  $R( )$  := being rectangular). The more

explicit operationalisation of the ‘being a cube of x’ is more complicated than of the ‘being the square of x’.

$K(\ )$  may be defined, for instance, also as ‘being squared’. This is becoming operational in a mathematical formula by  $K(x) := x*x$ . The multiplication sign ( $*$ ) is a mathematical verb (operator) for ‘multiplied by’ that was not yet explicit in  $K(\ )$ .

The symbol  $:=$  in this formula means ‘is defined as’ or ‘is per definition equal to’. So it is an operator as well, but it belongs to a meta-language *vis-à-vis* the terms at both sides of this operator. It has an essentially different meaning than the  $=$  sign (‘is equal to’ for calculations). By the same token the verb ‘is’ is ambiguous. That is the reason why well defined symbols originate making a distinction between  $:=$  and  $=$ . In the same vein there is a logical  $:\Leftrightarrow$  sign (‘is equivalent to’) that can be used to denote a logical equivalence, for example ‘is defined as’ or ‘is per definition equal to’.

In their turn functions are becoming only an assertion (the case or not the case) if the subject  $x$  has been substituted (for instance  $x :=$  ‘this building’). If a full-sentence function can be translated by substitution in an assertion that is the case, then it is ‘completable’.  $K(x):=x^2$  is completable for  $x \in \mathbb{R}$  ( $x$  as an element of the set of real numbers), but not for  $x \in$  of the set of buildings.

The full-sentence function may supply more than one subject with a predicate (here  $p$  and  $w$ ). It is then at home on several places. However, if one wants to include more predicates or within them more objects (not just buildings are dependent on population and prosperity, cars as well), there must also be more full-sentence functions. These can be conjoined with the linkage words from proposition logic, like  $\Rightarrow$  to an assertion like  $K(x) \Rightarrow R(x)$ .

### 23.10 A QUANTOR AS SUBJECT

In order to yield a meaningful assertion, the subject of a full-sentence does not need to be one concrete subject. Instead of defining  $x$  precisely one to one with reality (name giving) it can also be bound. This is particularly important to mathematics. Also  $\exists x$  (‘At least one  $x$ ’, existence-quantor) or  $\forall x$  (‘Each  $x$ ’, all-quantor) may yield a logically acceptable subject. In conjunction with the verb ‘:’ (‘satisfies’) and a full-sentence they may form an assertion. The assertion reads then, for instance, as  $\exists x : K(x)$ , ‘At least one building satisfies the description ‘cube’’ or  $\forall x : K(x)$ , ‘Each building satisfies the description ‘cube’’. The second, more general, assertion pre-supposes excellent scholarly breeding. However a generalising scholarly discipline is always looking for assertions with an all-quantor, since such a general assertion enables in a line of reasoning as a premise a wealth of deductive conclusions:

	<i>Formula</i>	<i>Read:</i>
1	$\forall x:(x \in X)$	For each $x$ ( $\forall x$ ) it is valid that (:): $x$ is an element from the set $X$ .
2	$\forall x:K(x)$	For each $x$ ( $\forall x$ ) it is valid that (:): $x$ is a cube.
3	$\forall x:(K(x) \Rightarrow R(x))$	For each $x$ ( $\forall x$ ) it is valid that (:): if $x$ is a cube, $x$ is also rectangular.
4	$\exists a:(a \in X)$	There is at least one $a$ ( $\exists a$ ) for which it is valid that (:): $a$ is an element of the set $X$ .
5	$\exists a:R(a)$	There is an ( $\exists a$ ) for which is valid that (:): $a$ is rectangular ( $R(a)$ ).

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Now we know that the second premise is not the case, if we substitute for  $x$  ‘building’ as an element of the set *all* buildings  $X$ . In order to get nevertheless a relatively general assertion, one must restrict the set, for instance to the set ‘buildings in this neighbourhood’  $B$ , if we know by complete induction that in this neighbourhood all buildings are cubes. Then it is sufficient to change the first premise into  $\forall x:(x \in X) \wedge (x \in B)$ . The symbol ‘ $\wedge$ ’ in this formula means ‘and’ in a sense well-defined in proposition logic.

### 23.11 THE CASE OR NOT THE CASE

Formal – mathematical feasible - logic, developed during previous centuries, is a more narrow notion than the concept ‘logic’ used to be in olden days. The word ‘logic’ is derived

from the word *'logos'*, a Greek word encompassing two illuminating clusters of meaning: speech itself, and giving account, testimonial. Logic as discussed here corresponds especially to the second cluster, as the lore of the right deductions.<sup>a</sup> The smallest possible 'im-mediate' deduction consists out of two propositions, separated by the two-letterword 'so': Holland is in The Netherlands, so The Netherlands are larger than Holland. More common is the 'mediate' deduction of a third proposition, a conclusion C, from two preceding premises A and B ('syllogism'), usually denoted as: A,B | C or

A	If it is winter, I am cold.	
B	It is winter	
		so
C	I am cold.	

Logic pre-supposes here, that propositions exist that may be the case, or not, but not both (yielding a contradiction) or both a little. This last restriction is removed in 'fuzzy logic', a branch of modern logic, disregarded in the following.

### 23.12 THE HUMAN POSSIBILITY TO DENY

A description of observations along these lines is only feasible, if we can imagine facts that are not the case. According to the Swiss psychologist Piaget, this capacity emerges in children when they are some eighteen months old. The capacity is hard to determine when it comes to animals, because they cannot express themselves to us in a way we can understand. Our brain must offer space to the not-here-and-now.

A filing cabinet should be ready there, as if it were, with the image 'it is winter', 'it is not winter', 'I am cold' and 'I am warm. As soon as something is the case, the box is full, as soon something is not the case, the box is empty. This has created space in our imagination for the true *and* false and thus for lies and deceit, but also for abstract thought and for the designing of things that are not (yet) there. Only with such an imaginative capacity (a 'logical space') at our disposal, can we arrive at rather general assertions like: 'If the sun starts shining, then I get warm'.

### 23.13 IF...

The following paragraphs provide an introduction into proposition logic on the basis of one of the most frequently used, and at the same time most confusing, logical operators, the word 'If...'. The 'if ... then ...' relation is of great interest to designers, since every design is an image of things that do not exist with an implicit promise: 'If you execute this, then you can dwell!'.

Compare the following assertions:

- 1 If it is winter, I am cold.
- 2 If it is winter, I could be cold.
- 3 If I am cold, then it is winter.
- 4 If I am cold, then it could be winter.
- 5 I only get cold if it is winter.
- 6 I am sometimes cold if it is winter.
- 7 I am always cold if it is winter
- 8 If it is winter, then I will probably be cold.
- 9 If it is winter, then you should turn on the heater, or else I will be cold.
- 10 I would like it to be winter because I am so warm.

The last expression is a wish, with on its background a lot of logical and causal pre-suppositions. The wish itself and its motivation do not belong to the linguistic game of logic, neither does the command (9) preceding it. In *both* expressions the hidden supposition "I will probably get warm" is a prediction or expectation that only becomes a fact, so true or false, if I really got warm. The grain of time is too small in the case to claim unambiguously a true or a false statement. Logic is necessary to arrive at such an expectation, but expectation itself

<sup>a</sup> More thorough introductions: Jong, W.R. de (1988) *Formele logika, een inleiding*; Eijck, J. van and E. Thijsse (1989) *Logica voor alfa's en informatici*; Sanford, D.H. (1989) *If P then Q, conditionals and foundations of reasoning*; Bentham, J.F.A.K. van, H.P. van Ditmarsch et al. (1994) *Logica voor informatici*.

surpasses the laws of logic. Assertions 1- 8 may be translated without complications into statements of the proposition-logical type.

### 23.14 STRESSING THE LOGICAL FORM

In order to study the type and its associated validity of the ‘if..then..’ relation as such, it is necessary that for the assertions used, any other assertion could be substituted without impairing the validity of the logical form itself.

If somebody makes an assertion, the logical investigation consists, therefore, especially in the search for counter-examples for which that type of the deduction becomes false. The assertions in the deduction are made variable then and the deduction gets the more abstract form ‘if p then q’.<sup>a</sup> To avoid possible confusion, we choose an example of the first assertion where the temporal aspect is absent:

‘If I am in Holland, then I am in The Netherlands’.

It is remarkable that this assertion can be ‘true’ if the partial statements are not the case, for instance, if I am in Hamburg. I am not in Holland then, not in The Netherlands, but *if* I am in Holland, I am also in The Netherlands, that stays ‘true’, even in Hamburg. It is also true when I am in Breda or of course, in Holland. The only case when I cannot uphold my assertion is when I am in Holland and it comes out that I am not in The Netherlands.

The truth-value of the assertion as a whole, depends this way on a specific combination of truth- values of the sub-statements ‘I am in Holland’ (P) and ‘I am in The Netherlands’ (Q). This may be summarised in a ‘truth-table’. There are four possibilities:

	I am in Holland P	I am in The Netherlands Q	Example	‘If P then Q’ $P \Rightarrow Q$	
1	Not the case	Not the case	Hamburg	True	184 If truth table
2	Not the case	The case	Breda	True	
3	The case	The case	Delft	True	
4	The case	Not the case	?	False	

### 23.15 DIFFERENT KINDS OF IF-STATEMENTS.

That was a clear example. But, if one returns to the old example ‘If it is winter, then I am cold’ and substitutes it, according to this table, I would be allowed to say ‘If it is not winter, then I am not cold’.

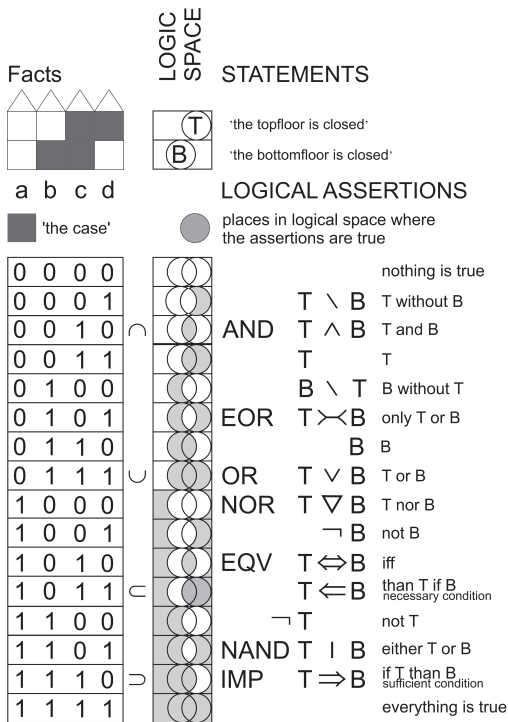
If someone has difficulty with that, it may be that he still values implicit causal pre-supposition.<sup>b</sup> It might also be that he envisages another ‘If...then..’ relation than the one above, to wit ‘If and only if’ (iff):

	It is winter P	I am cold Q	Example	Iff $P \Leftrightarrow Q$	
1	Not the case	Not the case	no winter, not cold	True	185 Iff truth table
2	Not the case	The case	no winter, cold	False	
3	The case	The case	winter, cold	True	
4	The case	Not the case	winter, not cold	False	

If we substitute this example again in that of paragraph 23.14 case 1 proves to be not to our liking. Suppose I am in Breda and say to a southerner ‘If I am in Holland, then I am in The Netherlands’. He is of the opinion, that I intend this reversibly and answers that that is not true, because I am not in Holland and yet in the Netherlands. Guilelessly, I mean the implication; he thinks that I mean the offending equivalence. I hope he knows the truth tables, for else this mis-understanding will never be sorted out.

a That is, by the way, also the title of a fine book on the history of logic: Sanford, D.H. (1989) *If P then Q, conditionals and foundations of reasoning*.  
b Hawkins, D.J.B. (1937) *Causality and implication*.

## propositional logic



logical assertions (facts) are combinations of statements (cases)

186 Complete truth table

187 If truth table

### 23.16 DISTINCTION BY TRUTH-TABLES

This shows how useful it is, that formal logic has developed different symbols (*implication*  $\Rightarrow$  and *equivalence*  $\Leftrightarrow$ ) and different logical operators for this confusing 'If.. then..' proposition. This distinction was possible by controlling the truth of the 'If P then Q' statement for each of the four states of affairs where  $P \in Q$  can be combined. For ' $\Rightarrow$ ' it turned out to be the sequence true, true, true and false (simplified by 1110), but for ' $\Leftrightarrow$ ' it turned out to be true, false, true and false (simplified by 1010).

Do the other combinations like 0000, 0001, 0010, etc. also mean something? It is easy to ascertain, that there are 16 such combinations that we can summarise in a table. A complete table like this appeared for the first time almost simultaneously shortly after the end of WW I in Wittgenstein's '*Tractatus*'<sup>a</sup> and with two other authors.

### 23.17 SUFFICIENT CONDITION

Just suppose, that four situations a, b, c and d are discerned for a residence under construction expressed in the combination of two assertions: 'The top floor has been provided with a façade' T, the bottom floor has been provided with a façade B, and the situation in which this is not the case. For these four cases a, b, c and d we verify now the assertion 'If the top floor has been provided with a façade, then the bottom floor has also been provided with a façade', crisply expressed as: ' $T \Rightarrow B$ ':

	Topfloor closed	Bottomfloor closed	Example	'If T then B'
	T	B		$T \Rightarrow B$
1	Not the case	Not the case	a	True
2	Not the case	The case	b	True
3	The case	The case	c	True
4	The case	Not the case	d	False

Only if the top floor has been provided with a façade, and the bottom floor not, we can not validate the assertion 'If the top floor has been provided by a façade, then the bottom floor has been provided with a façade as well'. So we have verified that ' $T \Rightarrow B$ ' is true for the first three cases, but not for the last case: (1110, 'sufficient condition').

### 23.18 EQUIVALENCE

Now, if a contractor is saying: 'If the top floor has been provided by a façade, *only* then also the bottom floor has been provided with a façade', case b is also invalid (1010, then and only then if, 'taoti', 'equivalence' iff).

	Topfloor closed	Bottomfloor closed	Example	'If T then B'
	T	B		$T \Leftrightarrow B$
1	Not the case	Not the case	a	True
2	Not the case	The case	b	False
3	The case	The case	c	True
4	The case	Not the case	d	False

188 Iff truth table

### 23.19 NECESSARY CONDITION

However, if the contractor is saying: 'Only if the top floor has been provided with a façade, then the bottom floor has been provided with a façade', then case d is suddenly valid again, but only case b not (1011, 'necessary condition').

	Topfloor closed	Bottomfloor closed	Example	'If T then B'
	T	B		$T \Leftarrow B$
1	Not the case	Not the case	a	True
2	Not the case	The case	b	False
3	The case	The case	c	True
4	The case	Not the case	d	True

189 Then ... if truth table

a Wittgenstein, L. (1922) *Tractatus logico-philosophicus*.



Each known logical operator like  $\Rightarrow$ ,  $\Leftrightarrow$  and  $\Leftarrow$ , for example ‘and’, ‘or’, ‘neither..nor’, ‘either..or’, proves to have a place on a truth-table (see diagram). Logical operators are more readily understood as equivalents of the set theoretical concepts  $\cap$ ,  $\cup$ ,  $\subset$ ,  $\supset$  or from drawings in which the sets are overlapping.

Symbolical rendering and its definition with the truth-table is now making an unambiguous distinction between the inclusive ‘or’ ( $\vee$ , OR) and the exclusive ‘either ...or’ ( $\>-<$ , EOR, XOR). The confusion of ‘and’ ( $\wedge$ ), and the inclusive ‘and’ in the sense of ‘and/ or’, the logical ‘or’ ( $\vee$ ) in daily parlance can not occur anymore. These logical operators should not be confused with sequential computer commands in an algorithm, such as the ‘IF...THEN...’ statement. That belongs to a different language game: the one of commands used for the execution of certain activities.

### 23.20 MODUS PONENS, TOLLENS AND ABDUCTION

In the examples below we assume that the implication ( $\Rightarrow$ ) is intended throughout.

We accept the following deduction:

- (1) If I am in Delft, then I am in The Netherlands.  
Well: I am in Delft  
----- So:  
I am in The Netherlands

We do not accept:

- (2) If I am in Delft, then I am in The Netherlands.  
Well: I am in The Netherlands.  
----- So:  
I am in Delft.

Yet we accept:

- (3) If I am in Delft, then I am in The Netherlands.  
Well: I am not in The Netherlands.  
----- So:  
I am not in Delft.

This seems obvious with examples directly connectible to enclosing sets (Delft is *in* The Netherlands), but why should we not accept (2) for example:

- (2\*) If it is winter, then I am cold.  
Well, I am cold.  
----- So:  
It is winter.

if we accept:

- (3\*) If it is winter, then I am cold.  
Well, I am not cold.  
----- So:  
It is not winter.

In these examples causal explanations are playing a confusing rôle. We know that the examples 2 and 2\* are logically not valid, but this line of reasoning is often used in medical practice, historiography, forming empirical hypotheses and in legal matters.

Suppose, a murder has been committed:

- (2\*\*) If X commits a murder, one finds his DNA  
Well, his DNA has been found  
----- So:  
X has committed the murder
- (3\*\*) If X commits a murder, one finds his DNA  
Well, his DNA has not been found  
----- So:  
X has not committed the murder

Examples 1 and 3 are known, respectively, as ‘*modus ponens*’ and ‘*modus tollens*’. Peirce has called the logically not-valid line of reasoning of form 2 ‘abduction’.<sup>a</sup> Abduction is used for finding a cause, even when one can never be sure of it.

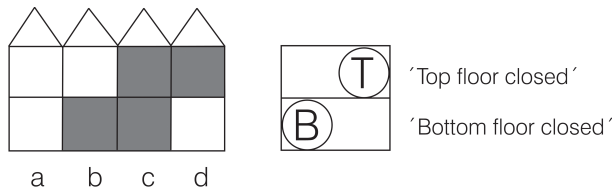
a Peirce, C.S. (1992) *Deduction, induction, and hypothesis*

### 23.21 VERIFYING LINES OF REASONING

We focus here on an example used previously in which no set theoretical or obvious causal connections are clashing with the logical connections.

- (1) If top floor closed, bottom floor closed  
 Top floor closed (T)  
 ----- So:  
 Bottom floor closed (B)
- (2) If top floor closed, bottom floor closed  
 Bottom floor closed (B)  
 ----- So:  
 Bottom floor closed (T)
- (3) If top floor closed, Bottom floor closed  
 Bottom floor not closed (not B)  
 ----- So:  
 Top floor closed (not T)

Again, we are distinguishing the following state-of-things (situations):



190 Three situations

and render the case and not the case with utterances, true and untrue with assertions both with respectively 1 and 0 in order to verify tree lines of reasoning:

	1 modus ponens			2 abduction			3 modus tollens		
	$T \Rightarrow B$	T	B	$T \Rightarrow B$	B	T	$T \Rightarrow B$	not B	not T
a	1	0	0	1	0	0	1	1	1
b	1	0	1	1	1	0	1	0	1
c	1	1	1	1	1	1	1	0	0
d	0	1	0	0	0	1	0	1	0

191 Modus ponens, tollens, abduction

For  $T \Rightarrow B$  the well-known substitution has been given, the assertions T, B, not T and not B have been derived from the drawing. Lines of reasoning are valid, when the premises and the conclusion are all true; or are 'the case' (1). With abduction there is a chance in situation b that conclusion T is not the case, even if both premises are true or the case.

So, one is not permitted to inter-change without damage premise and conclusion. A deduction is 'valid' when it is impossible to construct a counter example where the propositions are 'true', while the conclusion is 'false' (b). If one would accept that, any conclusion would be allowed.

### 23.22 INDUCTION

Lines of reasoning do not need to make use of an ‘if...then..’ operator. In the form of examples, we will now make use of the quantors and the ‘and’ operator in order to add a new form of reasoning. The first three of the following examples are known by now as deduction (1 and 3) and abduction (2). The fourth (4) was noted previously in page 191 as induction; although it is probably an incomplete induction here.

- (1) All houses in this neighbourhood are a cube  
This house is in this neighbourhood  
----- So:  
This house is a cube
- (2) All houses in this neighbourhood are a cube  
This house is a cube  
----- So:  
this house is in this neighbourhood
- (3) All houses in this neighbourhood are a cube  
This house is not a cube  
----- So:  
This house is not in this neighbourhood
- (4) This house is in this neighbourhood and is a cube  
Also this house is in this neighbourhood and is a cube  
Also this house is in this neighbourhood and is a cube  
----- So:  
All houses in this neighbourhood are a cube

For the first three forms of reasoning a general rule prevailed, but how to lay hands on such a rule? Example (4) enables this to happen by empirical induction. Since this is seldom complete, empirical science largely consists out of collecting samples. They must be statistically representative for the whole set studied in order to be able to draw a more general probable (not necessary) conclusion (generalisation). The tacit reasoning underlying this pre-supposition looks like an abduction. The more general rule may be used next in its turn in logically valid deductive forms of reasoning as a premise in order to make forecasts.

### 23.22 INNODUCTION

The example following does not belong to the logical language game, not even anymore to the language game of the empirical. The ‘But’ is marking an inductive part, the ‘So’ a deductive part. Without ‘But’ the reasoning is resembling abduction, but a negation has been inserted that yielded between (2) and (3) already a valid reasoning.

- (5) I am not warm.  
----- But:  
If I build a house, then I am warm.  
----- So:  
I build a house.

This line of reasoning is important to designing. It is a variant of *innoduction*.<sup>a</sup> A line and a new fact (in the original sense of ‘*factum*’, Latin for ‘made’) is added to the assertive premise ‘I am not warm’. The line between ‘But’ and ‘So’ is no premise and no conclusion in the classical sense of the word. It is a new idea and a pre-supposition, construed on occasion of and following from (and so not on an even ranking with) the asserting premise. There also could have stood ‘If a build a moderated microwave in my coat, then I’ve got it warm’. Although no premise, each change in this assertion is affecting the conclusion immediately. The ‘But’ signifies a shift in the passively asserting language game to an active pragmatic language game. It is introducing a negation of what is the case.

### 23.24 THE EMPIRICAL CYCLE

Now compare the following description (1), proposition (2), deduction (3) causal explanation. Do they have a 'logical form' in common with the world?

1. It is winter and I am cold.
2. If it is winter, I am cold.
3. It is winter, so I am cold.
4. It is winter, hence I am cold.
5. It is winter, but I am not cold.

In all these cases two propositions are connected: 'it is winter' and 'I am cold'. They have been connected with the word 'and', 'if...', 'so', 'hence' and 'but not', depending on the stage of our intellectual processing of our impressions (the 'empirical cycle', see page 249).

If I have experienced (1) repeatedly, I can conclude (2) for the time being. This kind of conclusion leads from specific statements to a more general one (induction). From this more general proposition, another specific statement (3) may be deduced (deduction). The third statement is an incomplete syllogism, since (2) is not mentioned. In the practice of language there is quite a lot not mentioned.

### 23.25 TACIT PRE-SUPPOSITIONS

Any reasoning lacks lots of premises, for example 'suppose we are human, suppose we have thoughts and a language to communicate, suppose you want to listen to me, suppose you do not kill me for what I say, suppose this building does not collapse, then I could tell you something'.

Culture contains a huge reservoir of unmentioned pre-suppositions. In the practice of language that is efficient, but it makes different cultures hard to understand. Making cultural pre-suppositions explicit is as hard as to get a description of water from a fish. The fish cannot compare its element with something else: for a description of the water, the possibility of its negation is necessary. Without difference, nothing can be perceived, chosen, described or thought.<sup>b</sup>

Also the general statement 'If it is winter, then I am cold', is only under certain pre-suppositions a fact, as long as we do not turn the heater on, put on warm cloths, take a warm shower, etc. Logic is oblivious of these conditions that are often so interesting to a designer by pre-supposing implicitly that the other circumstances stay equal (*ceteris paribus*).

### 23.26 PERCEPTION

The *expression* of a perception is closest to the 'world'; *facts* are perceived and expressed in a sentence. Consider the next example:

If the sun starts shining, I get warm	
The sun starts shining	
<hr/>	
I get warm	So:

According to Wittgenstein<sup>a</sup> the world is the totality of the *connections* (facts), not of the *things*. Basically I do not perceive the sun as a thing, but as a 'shining connection', for my first perception is 'something shines' (compare, 'something moves'), next I ask myself: 'What is that?' 'Something shines' can be rendered in formal logic as 'there is an x' ( $\exists x$ ) 'for which holds that' ( $:$ ) 'x shines' ( $S(x)$ ). Predicate logic codes that, like  $\exists x:S(x)$ . By the same token, shining is a function of x. It is still a variable, x: it may be a lamp or a sun, but it does shine. For convenience sake 'on me' is forgotten. That is not without importance, for it establishes a connection, a link. Next I can emancipate 'x shines' as 'the shining of x' that I can envelop by the function B 'beginning':  $\exists x:B(S(x))$ . Something starts shining, what is that? The sun:  $\exists z:B(S(s))!$  I have now substituted an independent name (s) of something that begins B shining S. What do I have gained here by substituting a noun? Is it not just the name, that other

a This term is suggested by Roozenburg, N.F.M. (1993) *On the pattern of reasoning in innovative design*. as an alternative for 'innovative abduction'. This term was suggested by Habermas for a form of abduction that was not explicit in Peirce. However the form of inroduction presented here does not co-incide with the form Roozenburg uses in his paper.

b Jong, T.M. de (1992) *Kleine methodologie voor ontwerp en onderzoek*.

people have given to the thing, as much as a naming function  $s=N(x)$ ? My formula extends:  $\exists x:B(S(N(x)))$ ; where is the end? What is named?

By perceiving this *connection* I can, for instance, distinguish the shining and the shone unto as active and passive things. Subsequently I can name these things with nouns, make them independent and use them as a subject 'Sun' and object 'that tree' or 'myself' as expressed in a sentence. Barring lies, the fact takes here from the world the barriers of the *impression* and the *expression* to land from that world into the sentence 'the sun is shining'. The fact that someone utters this full sentence is in its turn a new fact that has to take these hurdles again with other people.

A story to match can be told about the second statement 'I get warm' in spite of a number of new philosophical problems, like the meaning of the word 'I', the subjective experience of 'being cold', eventually as a 'property' of the 'I', the possible independence of the concept 'cold', etc. We leave those problems for what they are.

### 23.27 GRAIN

We assume that both perceptions have landed 'well' into the sentence, and that both are 'the case'; we consider both to be 'true'. They are two facts, combined by the word 'if ... then ...'. This little word establishes no causal connection like 'hence'; it just denotes that two facts on the same place and within a certain period ('here' and 'now') both are simultaneously 'the case'. That special condition is of importance, because of the local fact that the sun will shine somewhere else, the period imports, while the sun will set before too long. Each perception or observation implies place and time and a size of them both, the 'grain' of it.

In this case the grain was definitely smaller than half the surface of the earth and smaller than half of the 24 hours the earth takes for one spin around here axis, but larger than a point and a moment, since both do not have to occur simultaneously in an absolute sense, but for instance within the period of reliability of the assertion, a short time after one another. The under limit may be determined by asking across which area the observation was extended (in the second statement restricted to 'I'), and for how long the situation (the state of affairs) lasted.

The expression of the observation can now also be made more precise by indicating within the grain of time a sequence:

'First the sun starts shining and then I get warm'. Now suppose that it becomes cloudy next and that I'm getting cold. This expression of the facts is admittedly true, but I leave so many facts out of consideration (a 'half-truth'), that on the basis of this body of facts I can never arrive at a simple hypothesis, education or causal explanation that is known to us now.